Towards a Characterization of P Systems with Minimal Symport/Antiport and Two Membranes

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Key points of the systems definition

- Maximal parallelism
- Result at halting, in the elementary membrane

The power of some classes of P systems with small symport/antiport

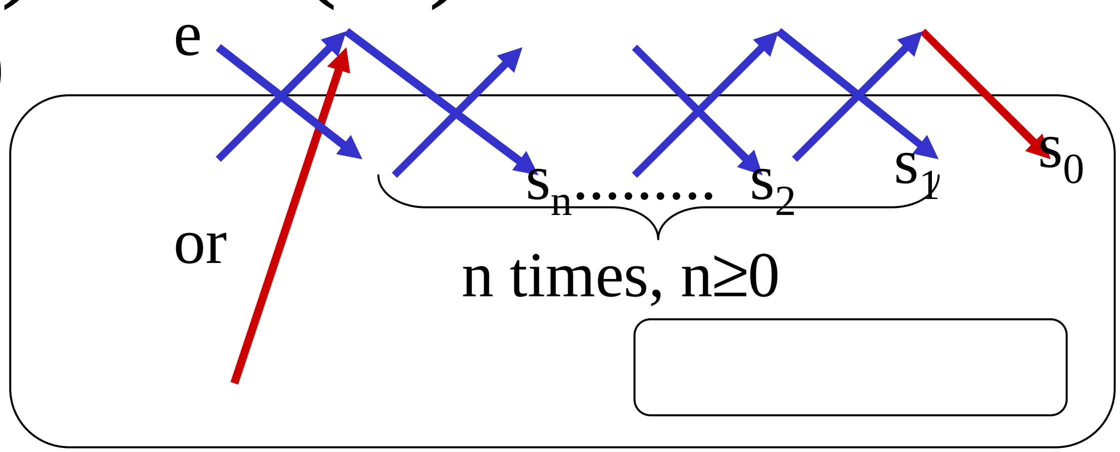
- $OP_3(sym_1, anti_1)$ and $OP_3(sym_2)$ are computationally complete

•$OP_2(sym_1, anti_1)$ and $OP_2(sym_2)$ are complete modulo some additional objects

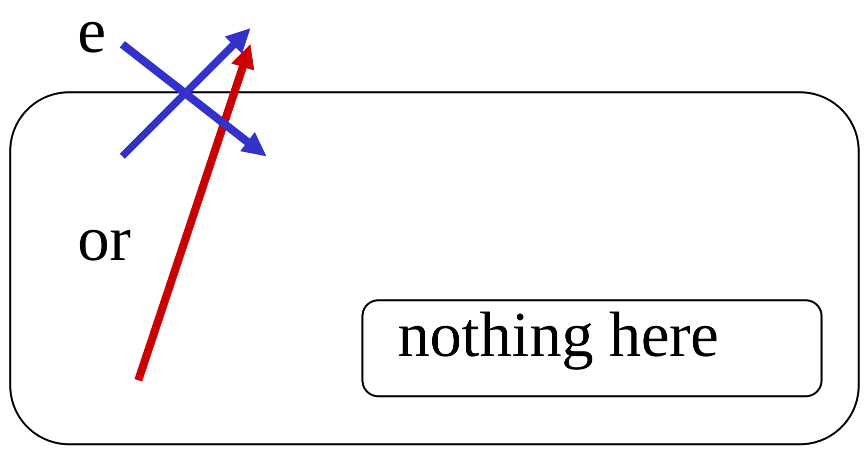
- $OP_1(sym_1, anti_{2/1})$ is computationally complete
- $OP_1(sym_3)$ is complete modulo 7 addditional objects

The Garbage is Unavoidable: If $\Pi \in OP_2(sym_1, anti_1)$ then $0 \in N(\Pi) \rightarrow N(\Pi) \in NFIN$

- Suppose $N(\Pi)$ is infinite

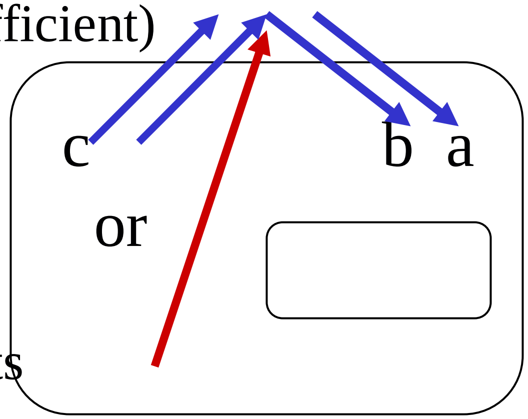


- Suppose $N(\Pi)$ contains 0: it halts
- s_0 is in region 1=> so is $s_1, \ldots, s_n$
- contradiction



Symport Garbage: If $\Pi \in OP_2(sym_2)$ then $0 \in N(\Pi) \rightarrow N(\Pi) \in NFIN$

- Call I_0 the set of objects from O-E that we know must be in the environment at halting with empty region 2; $I_0 := \emptyset$.
- Assume $N(\Pi)$ is infinite: it is necessary (though not sufficient) to bring in some object $a \in E \cup I_0$: $(ab,in) \in R_1$ ($b \in$ O-E).



•(b,out): cannot halt with empty region 2.
•(bc,out), $c \in$ O-E: c does not stay in the environment (otherwise it does not help increasing the number of objects inside the system).
•Still, c cannot end up in region 1 if b is there. Add c to I_0 and repeat.
•(bc,out), $c \in E$: repeat with c instead of a.

•If a system can increase the number of objects inside it, then it cannot halt without any objects in region 2.

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Notations $: N_k RE = \{k + L \mid L \in NRE\},$

$N_1 RE = \{N \in NRE \mid 0 \notin N\}.$

$N_{\ni 0} FIN = \{N \in NFIN \mid 0 \in N\},$

$N_{\ni 0} SEG_1 = \{\{k \in N \mid k < n\} \mid n \geq 0\}.$

Theorem 3. $NOP_2(sym_1, anti_1) = N_1 RE \cup F$, where $N_{\ni 0} SEG_1 \subseteq F \subseteq N_{\ni 0} FIN$.

Proving only $N_1 OP_2(sym_1, anti_1) = N_1 RE$

Outline of the proof

- We simulate a d -counter automaton

$$M=(d, Q, q_0, q_f, P).$$

$Q=\{q_i \mid 0 \leq i \leq f\}$ states, q_0 initial, q_f final,

P finite set of instructions.

"increment" j: ($q_i \rightarrow q_l$,k+)

"decrement" j: ($q_i \rightarrow q_l$,k-)

"test for zero" j: ($q_i \rightarrow q_l$,k=0)

Construction

Notations: $C=\{c_k\}$, $k\in\{1,\ldots,d\}$, $Q'=\{q_i'\}$, $q_i\in Q$.

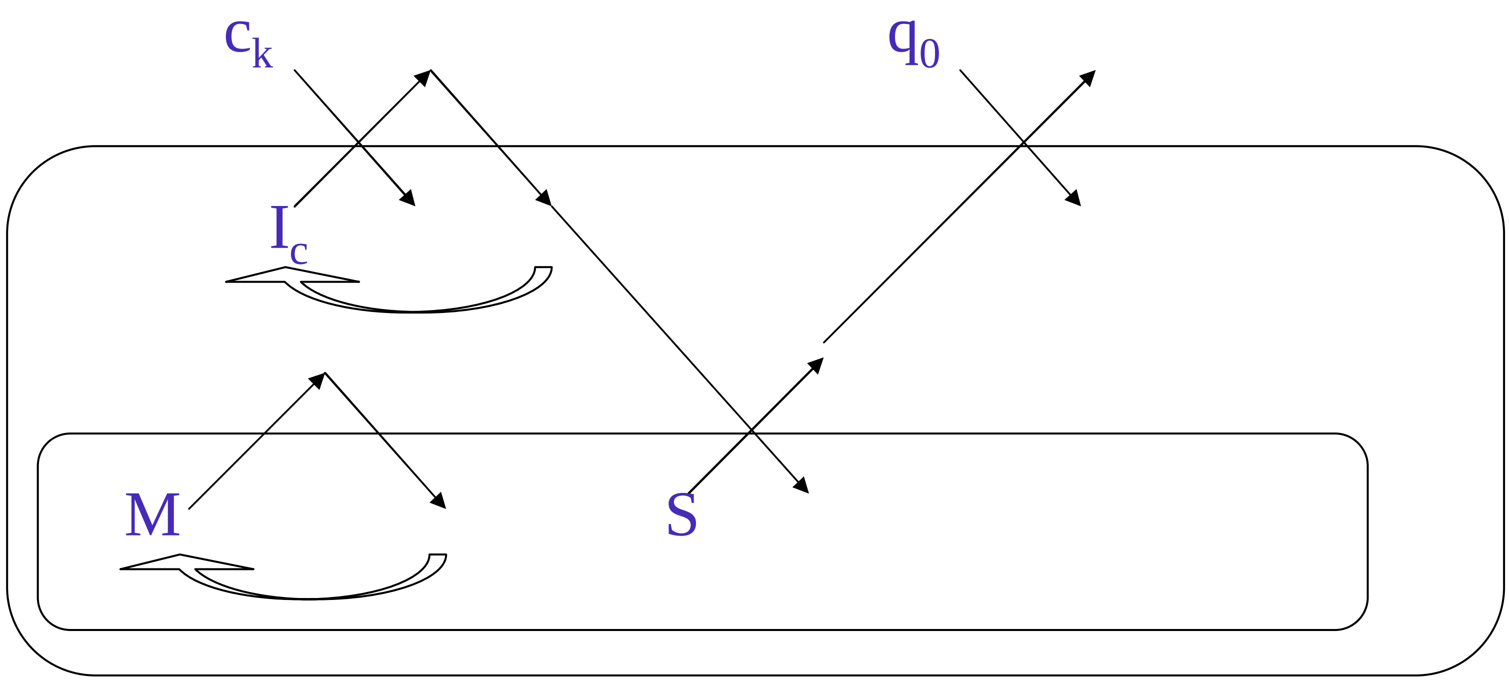
We construct the P system Π_1 as follows:

$$\Pi_1 = (O, [_1\ [_2\ ]_2\ ]_1, w_1, w_2, E, R_1, R_2, 2),$$
$$O = E \cup \{I_c, M, S, T_1, T_2, J_1, J_2\} \cup \{b_j, d_j \mid j \in I\},$$
$$E = Q \cup Q' \cup C \cup \{a_j, a_j' \mid j \in I\} \cup \{J_0, F_1, F_2, F_3, F_4, F_5\},$$
$$w_1 = J_1 J_2,$$
$$w_2 = T_1 T_2 M S \prod_{j\in I} b_j \prod_{j\in I} d_j,$$

The functioning of this system may be split into three stages:

- preparation of the system for the computation.
- simulation of instructions of the counter automaton.
- terminating the computation.

Preparation



Main stage: representation

supply of states

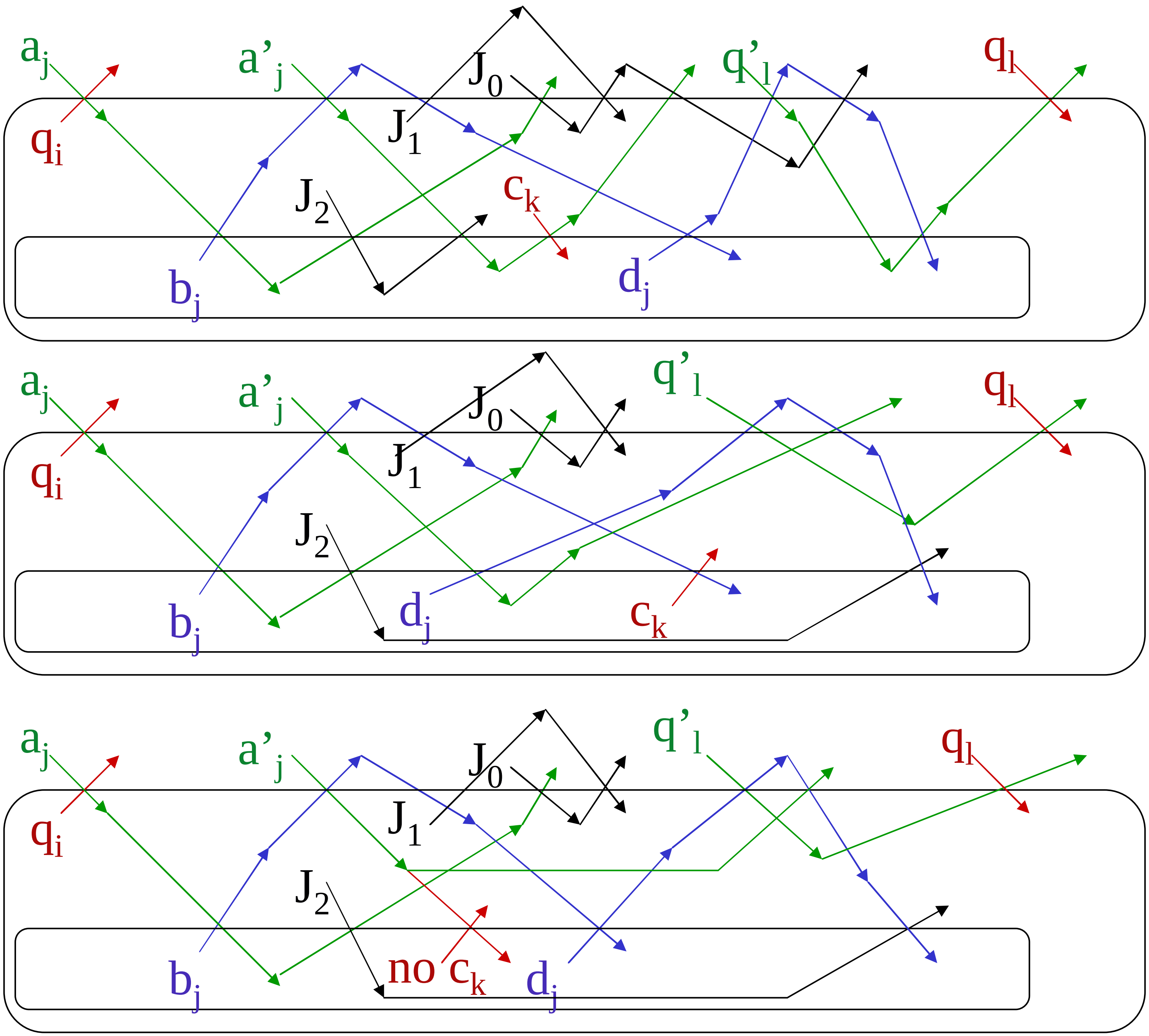
supply of instructions

q_i :current state

supply of counters

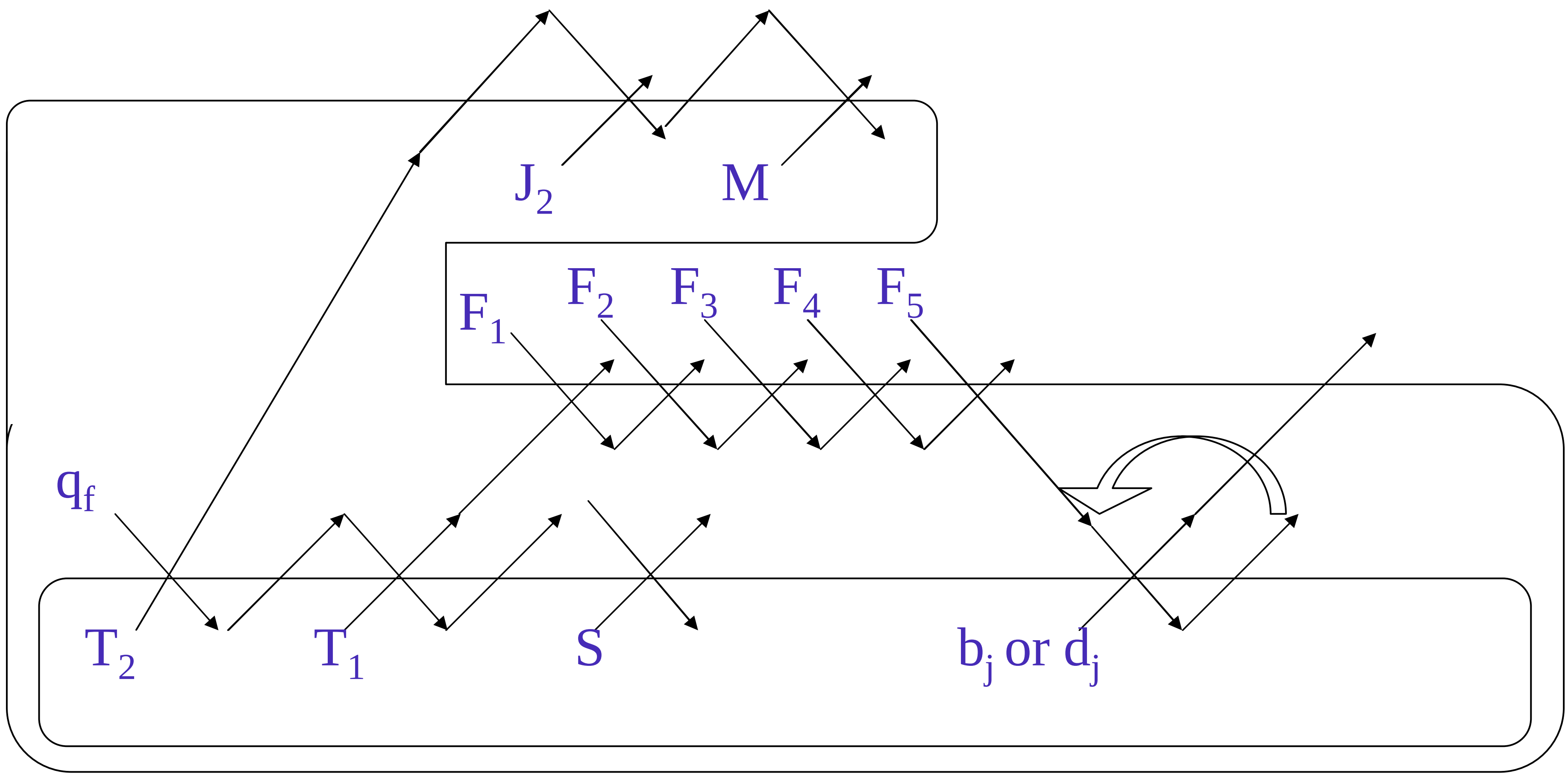
c_k :counters

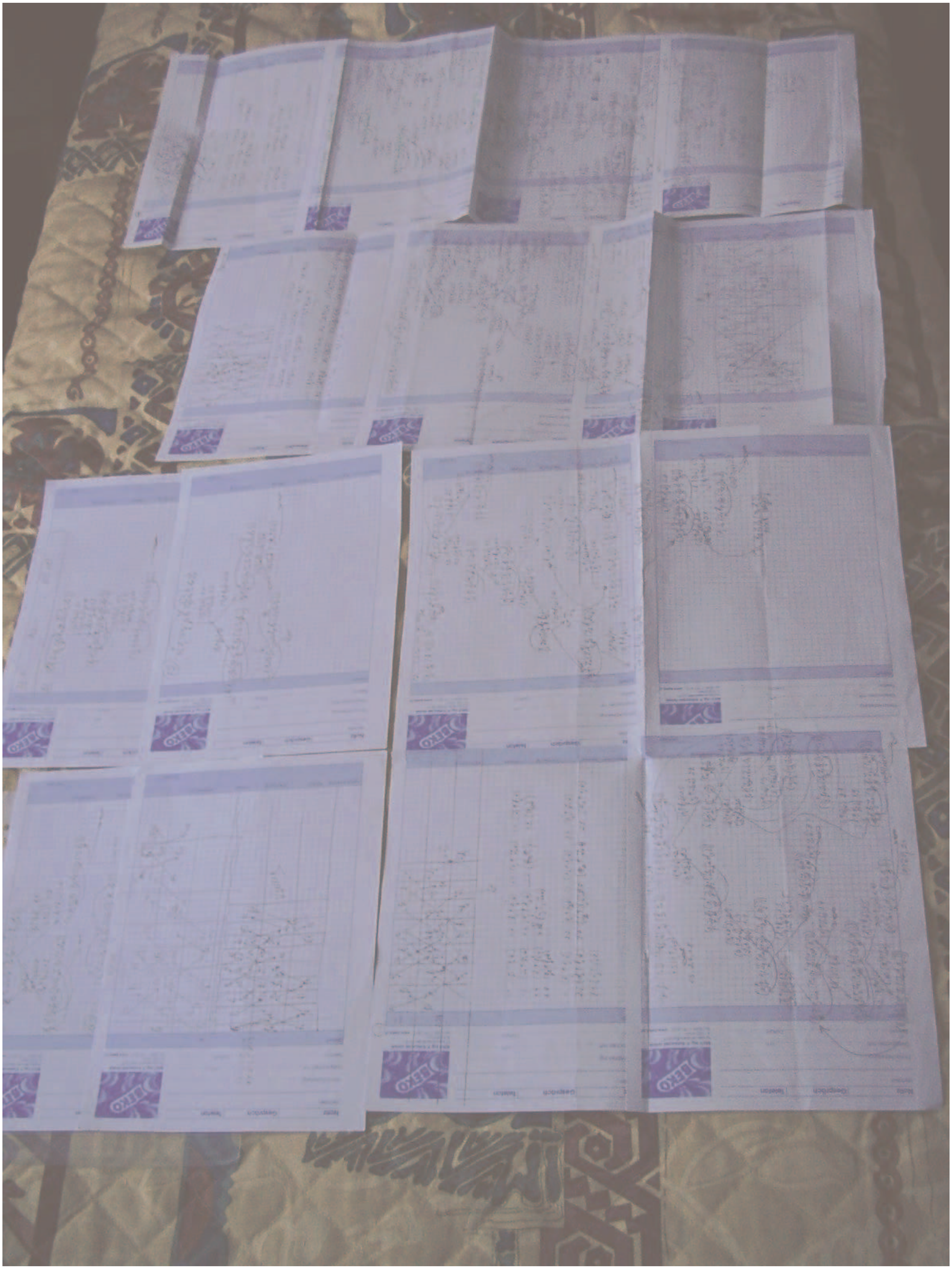
Main stage: +,-,=0



a_j
q_i
a'_j
J_0
J_1
J_2
b_j
c_k
d_j
q'_l
q_l
a_j
q_i
a'_j
J_0
J_1
J_2
b_j
d_j
c_k
q'_l
q_l
a_j
q_i
a'_j
J_0
J_1
J_2
b_j
no c_k
d_j
q'_l
q_l

Termination





Conclusion

- P systems with minimal symport/antiport and two membranes: the optimal result is obtained.
  - One additional object in the output membrane is necessary and sufficient for computational completeness.
- For P systems with two membranes and symport of weight 2,
  - one object is necessary; sufficiency is open.
- It is still open what finite sets containing zero can be generated by these systems
  - Conjecture: $\{\{m|m\leq n\}n\in N\}$