Graphical Modelling of Higher Plants Using P Systems

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Goal

Modelling developmental processes of plants

- Simple developmental algorithms
- Self-similarity
- L-systems with geometric features
Outline

1. Lindenmayer Systems
2. Graphical Representation of Lindenmayer Systems
3. Restricted P Systems with Membrane Creation
4. Graphical Representation of Restricted P Systems with Membrane Creation
5. Final Remarks
String OLL-system

\[ G = \langle V, \omega, P \rangle \]

- \( V \) alphabet
- \( \omega \in V^+ \) axiom
- \( P \) production rules

Production rule: \( a \rightarrow v \)

- \( a \in V \) predecessor
- \( v \in V^* \) successor
- For any \( a \in V \) at least one \( a \rightarrow v \in P \)
Evolution of L-Systems

Parallel rewriting

\[ \rho \in V^* \text{ directly derives from } \mu \in V^+: \mu \Rightarrow \rho \]

- \[ \mu = a_1 \ldots a_m, \text{ with } a_i \in V \]
- \[ \rho = \phi_1 \ldots \phi_m, \text{ with } \phi_i \in V^* \]
- \[ a_i \rightarrow \phi_i \in P, \text{ for all } i = 1, \ldots, m \]

Sequence of strings \( \omega, \mu_1, \mu_2, \ldots \) generated recursively
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Workshop on Membrane Computing 2006
Turtle graphics

Step size and turn angle fixed

Graphical commands:

- $F$: move a step forward drawing a line
- $f$: move a step forward not drawing a line
- $+$: turn left
- $-$: turn right
L-System Representation: Example

Step size: 2 cm  
Turn angle: 60 degrees

\[ F + F -- F + F \]
Consider a push-down stack.

Additional graphical commands:

- [: push current state onto the stack
- ]: pop a state from the stack
Tree Structure Representation: Example

Step size: 2 cm    Turn angle: 22.5 degrees

\[ F[+F[+F][F]][F][F][−F] ] \]
Overall Picture

Strong points:
• Simple
• Deeply studied
• Many software available

Drawbacks:
• Artificial
• Unrealistic
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Membrane Structures Are Trees
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![Diagram of membrane structures and a tree structure]
Membrane Structures Are Trees
Membrane Structures Are Trees
Basic Requirements

Looking for a model:

- As simple as possible
- With membrane creation

Drop from P systems with membrane creation everything not needed
Restricted P Systems with Membrane Creation

\[ \Pi = (O, \mu, w_1, \ldots, w_m, R) \]

- \( O \) alphabet of objects
- \( \mu \) initial membrane structure
- \( w_i \) multiset initially placed in region \( i \)
- \( R \) finite set of evolution rules:
  - \( a \rightarrow v \), with \( a \in O \), \( v \) multiset over \( O \)
  - \( a \rightarrow [v] \), with \( a \in O \), \( v \) multiset over \( O \)

A unique label for the membranes, so \( R \) global set of rules
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Graphical Model for P Systems

Branch length and rotation angle fixed

Depth-first search of the membrane structure

Graphical objects:

- $F$: draw the branch
- $+$: rotate the branch to the left
- $-$: rotate the branch to the right
Π₁ components:

- Alphabet: \( \{ F, +, -, B_L, B_R, B_{S_1}, B_{S_2} \} \)
- Initial membrane structure and multiset:
  \( [FB_L B_{S_1}] \)
- Rules:

\[
\begin{align*}
B_{S_1} & \rightarrow [FB_{S_2} B_R] & B_L & \rightarrow [+FB_L B_{S_1}] \\
B_{S_2} & \rightarrow [FB_L B_{S_1}] & B_R & \rightarrow [-FB_L B_{S_1}] 
\end{align*}
\]
P System Representation: Example 1

Branch length: 2 cm    Rotation angle: 22.5 degrees
P System Representation: Example 1

Branch length: 2 cm       Rotation angle: 22.5 degrees
P System Representation: Example 1

Branch length: 2 cm
Rotation angle: 22.5 degrees
Extended Graphical Model for P Systems

Fix lengths \( l \) and \( w \) and angle \( \delta \)

Graphical objects: \( F \), \( W \), + and −

For each membrane draw a branch with:

- Length: \((\text{multiplicity of } F) \times l\)
- Width: \((\text{multiplicity of } W) \times w\)
- Rotation angle:
  \((\text{multiplicity of } + \text{ minus multiplicity of } -) \times \delta\)
$\Pi_2$ components:

- **Alphabet:** \( \{ F, W, +, -, L, E, B_L, B_R, B_{S_1}, B_{S_2} \} \)
- **Initial membrane structure and multiset:**
  \[ [LEWFB_LB_{S_1}] \]
- **Rules:**

  \[
  \begin{align*}
    B_{S_1} & \rightarrow [LEWFB_{S_2}B_R] \\
    B_{S_2} & \rightarrow [LEWFB_LB_{S_1}] \\
    L & \rightarrow LF \\
    B_L & \rightarrow [+LEWFB_LB_{S_1}] \\
    B_R & \rightarrow [-LEWFB_LB_{S_1}] \\
    E & \rightarrow EW
  \end{align*}
  \]
P System Representation: Example 2

$l$: 1 cm     $w$: 1 pt     $\delta$: 22.5 degrees

\[ LEWFB_L B_{S_1} \]
P System Representation: Example 2

$l$: 1 cm  \( w \): 1 pt  \( \delta \): 22.5 degrees

\[ LEW^2F^2 \]
\[ LEWFB_{S_2}B_R \]
\[ +LEWFB_{L}B_{S_1} \]
P System Representation: Example 2

\[ l: 1 \text{ cm} \quad w: 1 \text{ pt} \quad \delta: 22.5 \text{ degrees} \]
Specimen-to-Specimen Variation

Stochastic P systems: associate each rule with a probability

\( \Pi_3 \) components:

- Alphabet: \( \{F, W, +, -, L, E, T, B\} \)
- Initial membrane structure and multiset: \([LEWFTB]\)
- Rules:

\[
\begin{align*}
T & \rightarrow [LEWFTB] & L & \rightarrow LF & E & \rightarrow EW \\
B & \xrightarrow{2/3} [+LEWFTB] & B & \xrightarrow{1/3} [-LEWFTB]
\end{align*}
\]
Trees generated by $\Pi_3$

$l: 1$ cm $\quad w: 1$ pt $\quad \delta: 22.5$ degrees
Trees generated by $\Pi_3$

$l$: 1 cm \hspace{1cm} w:1 pt \hspace{1cm} \delta$: 22.5 degrees
Trees generated by $\Pi_3$

$l$: 1 cm  $w$: 1 pt  $\delta$: 22.5 degrees
Specimen-to-Specimen Variation

Non-deterministic P systems: consider together all the trees generated by its computations

\( \Pi_4 \) components:

- Alphabet: \( \{ F, W, +, -, L, E, T, B \} \)
- Initial membrane structure and multiset: \([LEWFTB]\)
- Rules:

\[
\begin{align*}
T & \rightarrow [LEWFTB] \quad L \rightarrow LF \quad E \rightarrow EW \\
B & \rightarrow [+LEWFTB] \quad B \rightarrow [-LEWFTB]
\end{align*}
\]
Trees generated by $\Pi_4$

$l: 1\ cm \quad w: 1\ pt \quad \delta: 22.5\ degrees$
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P Systems versus L-Systems

Two things to investigate:

- Complexity of the representation of the branching structures
- Computational efficiency to generate the graphical representations
Conclusions

Strong points of P systems:

- Closer to reality
- Supports differentiation into small units
- Computational power

Drawbacks:

- No software available (¿Translations to L-systems?)
- Parsing algorithm more complex
Extensions of the model

- Labelling of the membranes
- Communication rules
- Rules of the form $o \rightarrow \mu$, with $\mu$ a membrane structure
Thanks for your attention