A Case Study in (Mem)Brane Computation: generating $\{n^2 \mid n \geq 1\}$

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Aim of the work

- Long term goal:
  - polynomial solution to NP-complete problems in Brane Calculi?
- Aim of the paper:
  - compare P systems and Brane Calculi w.r.t. their ability to act as language generators

Overview

- P systems and Brane Calculus
- Generating $\{n^2 \mid n \geq 1\}$ in P systems
  - "traditional" solution
  - Simplified solution
- Brane calculus
- Generating $\{n^2 \mid n \geq 1\}$ in Brane Calculus
- Conclusion

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Models for membranes

- P systems [Paun00]
  - Automata and formal languages theory
  - Computational power of the living matter
  - Turing completeness, complexity
- Brane calculi [Cardelli04]
  - Process calculi, concurrency theory
  - Tool for systems biology
  - Use concurrency theory to analyse the living matter

Bridging the gap

- Brane Calculi as source of inspiration for new classes of P systems with brane-like operations
  - [CardelliPaun05, PaunPopa06, Cavaliereetal06, Besozzietal0, Proc. MeCBIC 2006, Proc. WMC7]
- Compare the expressivity of Brane Calculi and P systems
  - Universality results for classes of Brane Calculi [BusiGorrieri05, Busi06]
  - Brane Calculi and P systems as language generators

Key differences

- Active entities
  - (classical) P systems: objects (floating molecules) are inside membranes
  - Brane Calculi: processes (membrane proteins) are on membranes
- Semantics
  - Maximal parallelism for P systems
  - Interleaving (sequential) for Brane Calculi

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P systems: rules

- Object evolution: $[a \rightarrow v]_h$
- Cooperation: $[u \rightarrow v]_h$
- Dissolution: $[a]_h \rightarrow b$
- Send-in comm.: $a[\ ]_h \rightarrow [b]_h$
- Send-out comm.: $[a]_h \rightarrow b[\ ]_h$

Generating $\{n^2 \mid n \geq 1\}$

- First solution [Paun02]:
  - (object evolution), cooperation, send-out, dissolution
  - Priorities
- Simpler solution:
  - (object evolution), send-in, dissolution

Generating $\{n^2 \mid n \geq 1\}$: simpler solution

- Idea: $\forall n \geq 0 : (n+1)^2 = \sum_{k=0}^{n} (2k+1)$
- Membrane structure: $[\ [\ ]_e\ [\ ]_r\ ]_s$
- Initial state: $[\ [a^2bz]_e\ [\ ]_r\ ]_s$
- Output: number of objects c in membrane r in a halting computation

Generating $\{n^2 \mid n \geq 1\}$: rules

1. $[a \rightarrow ab]_e$
2. $[b \rightarrow bc]_e$
3. $[z \rightarrow z]_e$
4. $[z]_e \rightarrow \lambda$
5. $[a \rightarrow \lambda]_s$
6. $[b \rightarrow \lambda]_s$
7. $c[\ ]_r \rightarrow [c]_r$

Generating $\{n^2 \mid n \geq 1\}$: nondeterminism

1. $[a \rightarrow ab]_e$
2. $[b \rightarrow bc]_e$
3. $[z \rightarrow z]_e$
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6. $[b \rightarrow \lambda]_s$
7. $c[\ ]_r \rightarrow [c]_r$

Continue computation to generate $(n+1)^2$

Reach an halting computation with output n^2

Generating $\{n^2 \mid n \geq 1\}$: evolution

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$[\ [a^2b^3cz]_e\ [\ ]_r\ ]_s$

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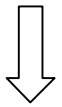

$[\ [a^2b^3cz]_e\ [\ ]_r\ ]_s$

$[\ [a^2b^5c^4z]_e\ [\ ]_r\ ]_s$

Generating $\{n^2 \mid n \geq 1\}$: evolution

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⇩

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⇩

$[\ [a^2b^5c^4z]_e\ [\ ]_r\ ]_s$

...

$[\ [a^2b^{2n+1}c^{n\hat{}2}z]_e\ [\ ]_r\ ]_s$

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⇩

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⇩

$[\ [a^2b^5c^4z]_e\ [\ ]_r\ ]_s$

...

$[\ [a^2b^{2n+1}c^{n\hat{}2}z]_e\ [\ ]_r\ ]_s$

⇩

$[\ [a^2b^{2(n+1)+1}c^{(n=1)\hat{}2}z]_e\ [\ ]_r\ ]_s$

Generating $\{n^2 \mid n \geq 1\}$: evolution

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$[\ a^2b^{2(n+1)+1}c^{(n+1)\hat{}2}\ [\ ]_r\ ]_s$

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⇩

$[\ a^2b^{2(n+1)+1}c^{(n+1)\hat{}2}\ [\ ]_r\ ]_s$

⬇

$[\ [\ c^{(n+1)\hat{}2}]_r\ ]_s$

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Brane calculus: membrane interactions

- Endocytosis/Exocytosis
  - Incorporate external material into a cell by engulfing it with the cell membrane
  - Reverse operation
- Mate/Mito
  - Membrane fusion/fission

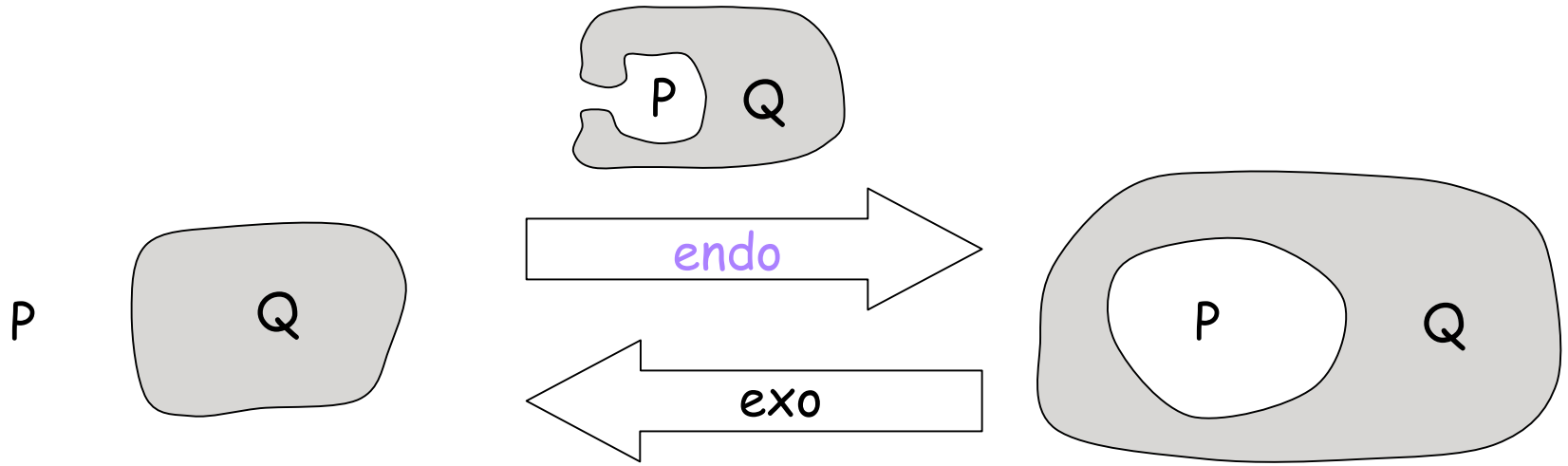
Brane Calculi: features

- Hierarchy of nested branes
- Evolution of the hierarchical structure driven by membrane interactions
- **Interleaving (sequential) semantics**
- **Active entities reside on branes** (membrane proteins) and …
- … may perform interaction on **both** sides of the brane

Basic Brane Calculi

- Only membranes and membranes interactions
- No molecules
- Two sets of interaction primitives [Cardelli]
  - Phago/Exo/Pino (PEP)
  - Mate/Bud/Drip (MBD)

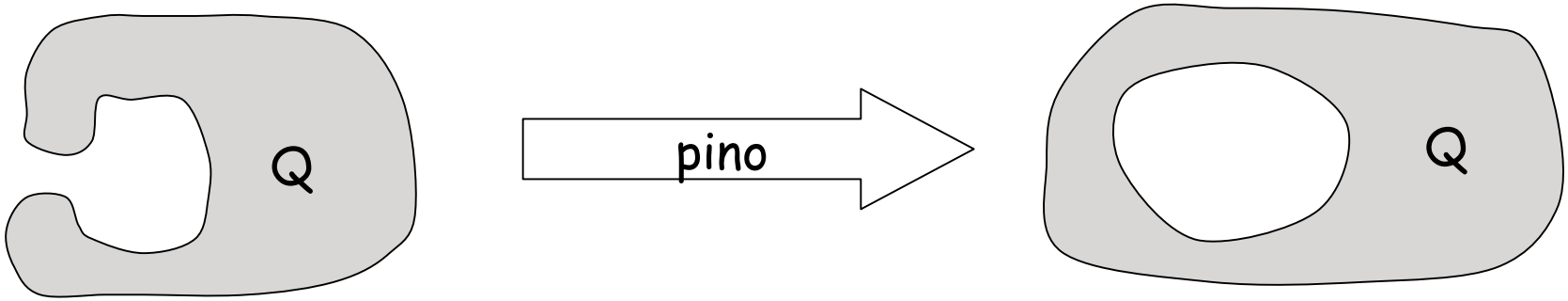
Endo/exo interactions



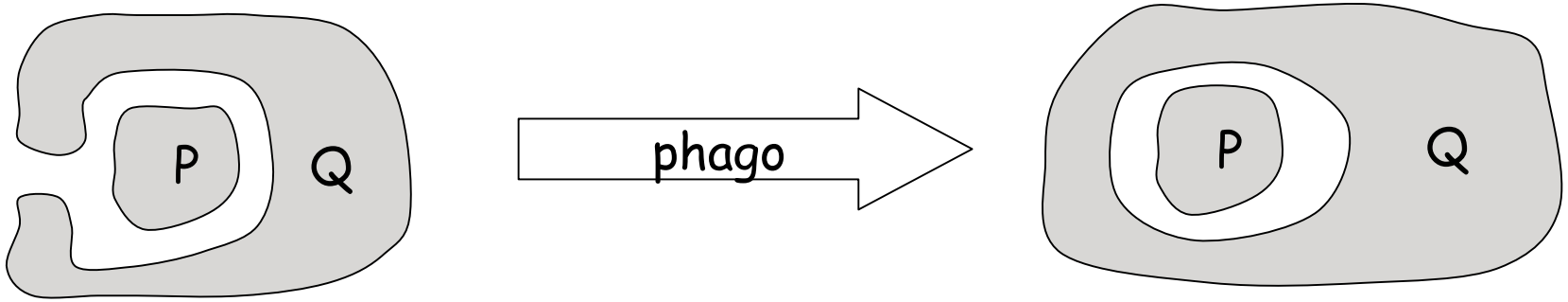
- Bitonality: no mix of material inside the brane with material from outside
- Endocytosis can engulf an arbitrary number of entities - uncontrollable process

Endocytosis: special cases

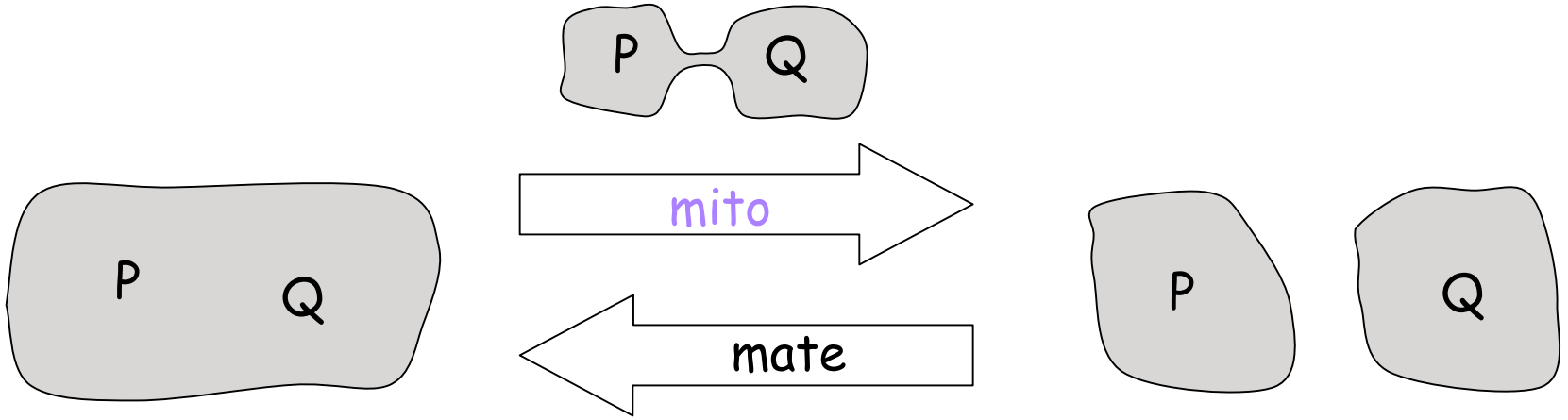
Zero case:



One case:



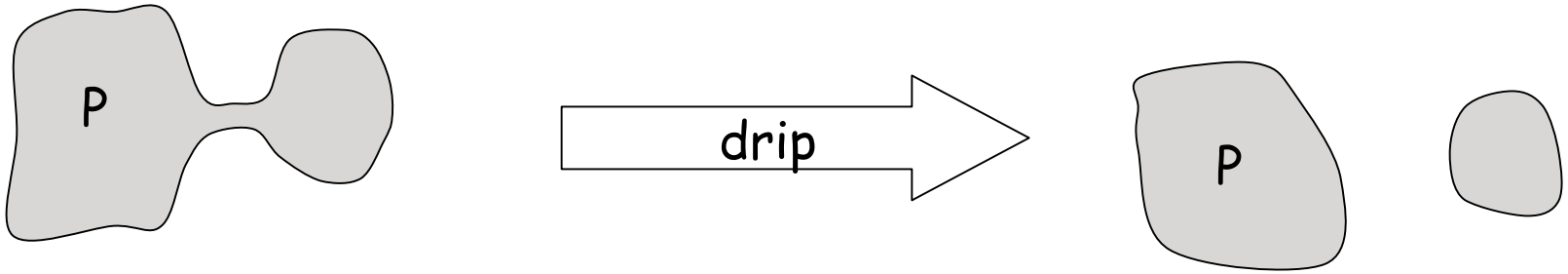
Mate/mito interactions



- Mito is an uncontrollable process

Mito: special cases

Zero case:



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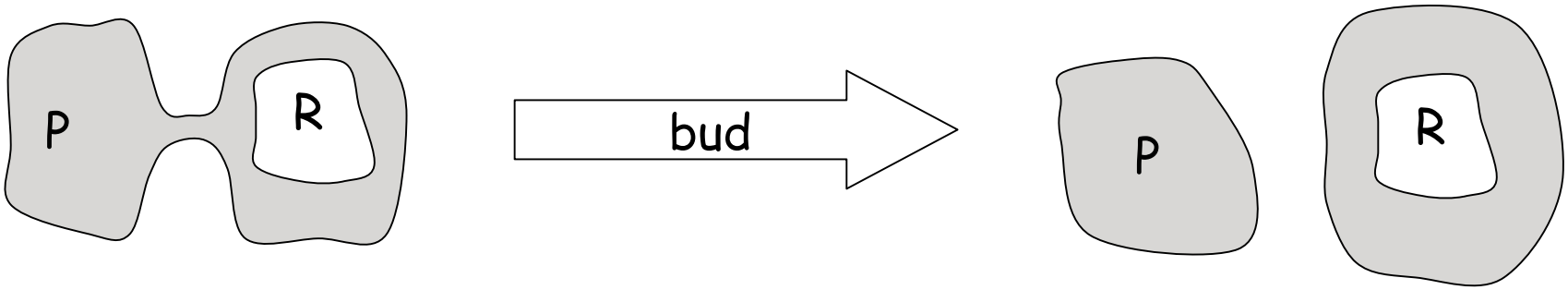
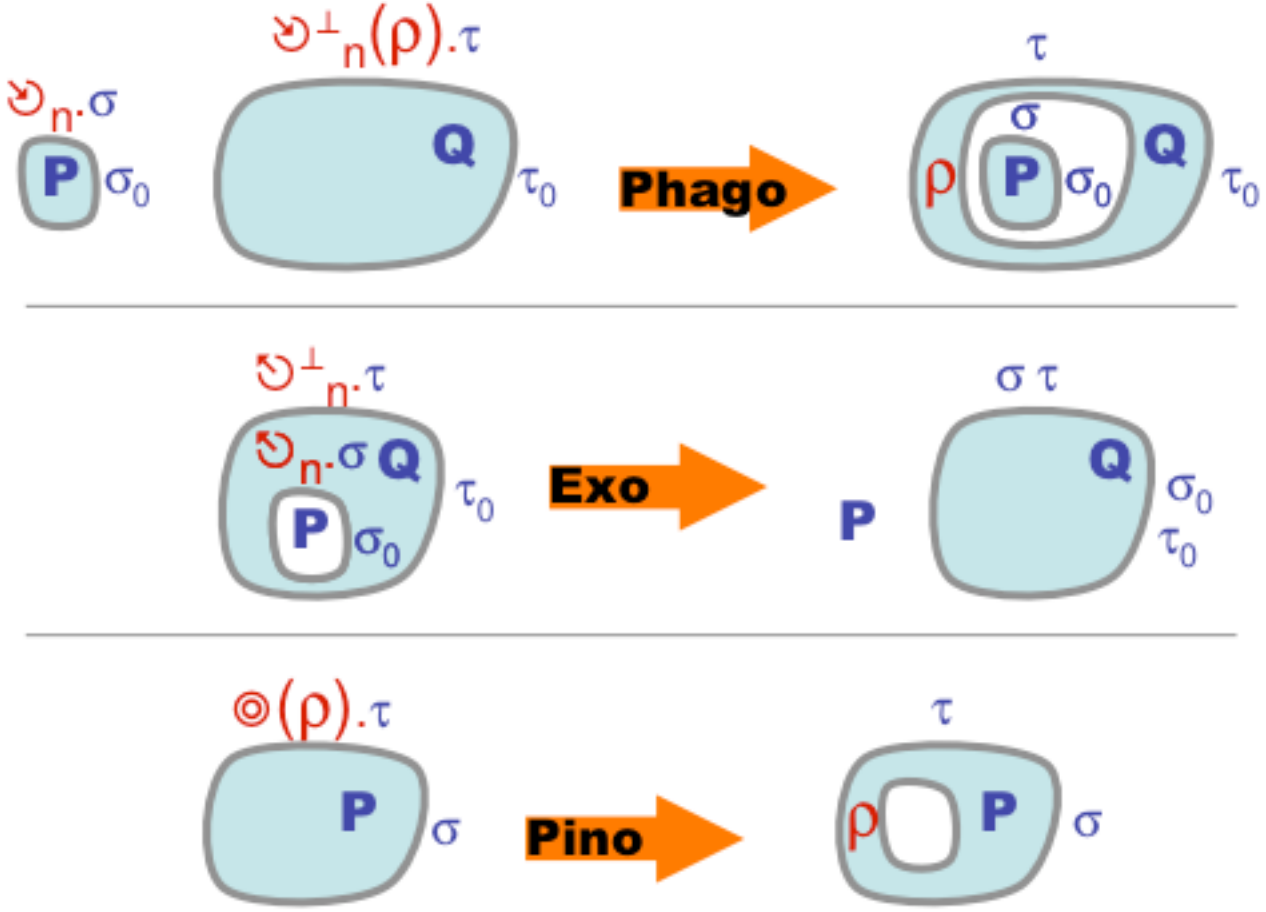
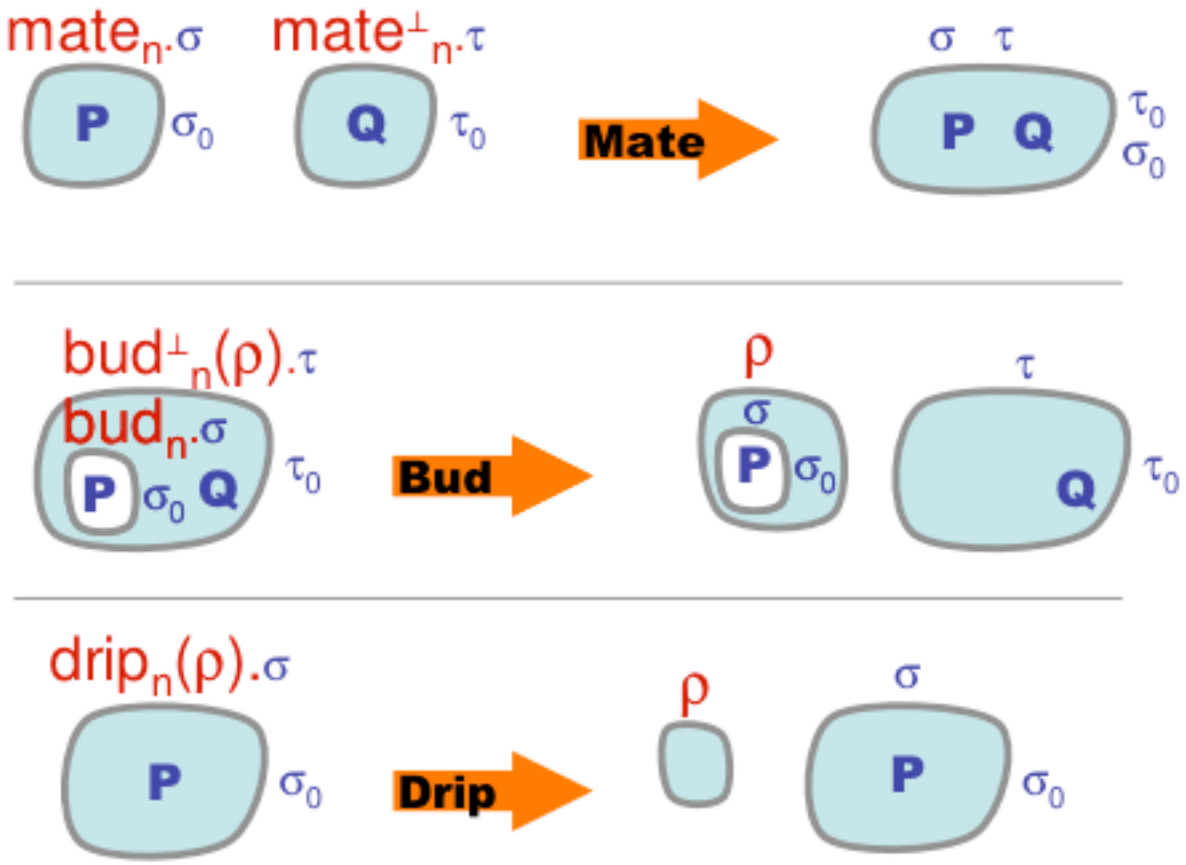


Semantics of PEP



[Cardelli]

Semantics of MBD



[Cardelli]

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Generating $\{n^2 \mid n \geq 1\}$ in Brane Calculus: output

- P system: number of objects c in membrane r in a halting computation
- Brane calculus: number of membranes with process C on them
  - Scattered in the whole system
  - Contained in a Res membrane

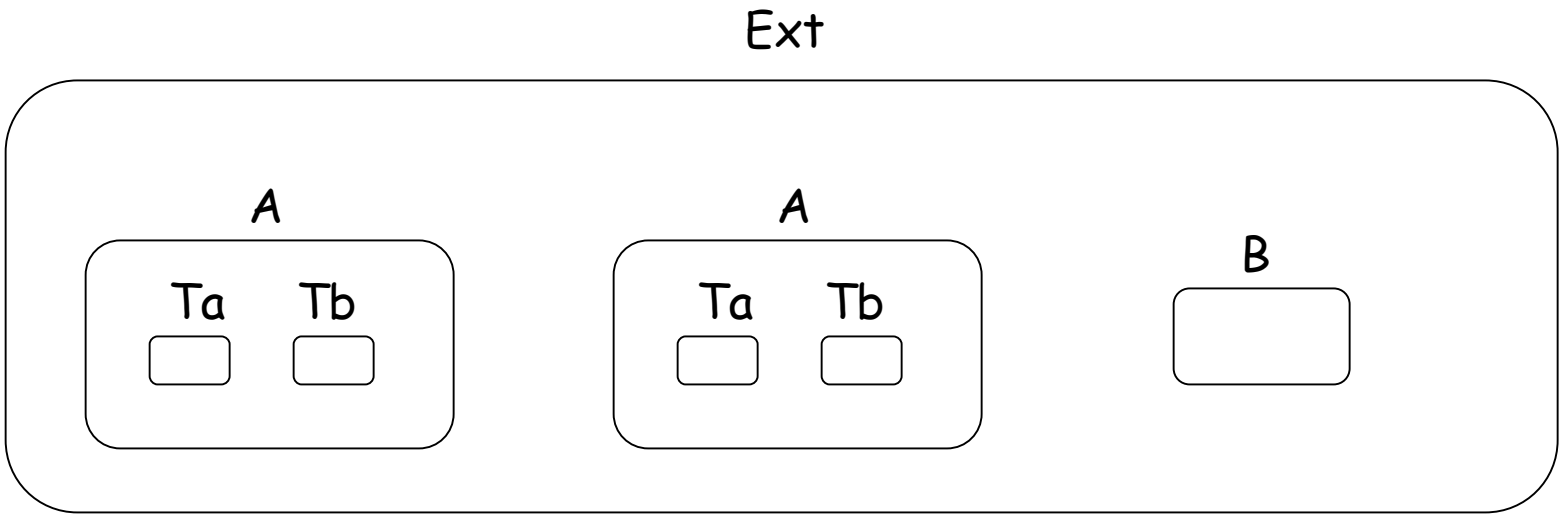
Generating $\{n^2 \mid n \geq 1\}$ in Brane Calculus: semantics

- Maximal parallelism: very powerful synchronization mechanism
- Naïve solution: encode object a with membrane A[ ] etc:
  - S[A[ ] A[ ] B[ ]]
  - Interleaving semantics: no guarantee that all the applicable rules are applied
- We need to synchronize an unbounded number of branes:
  - Very difficult task

Generating $\{n^2 \mid n \geq 1\}$ in Brane Calculus: semantics

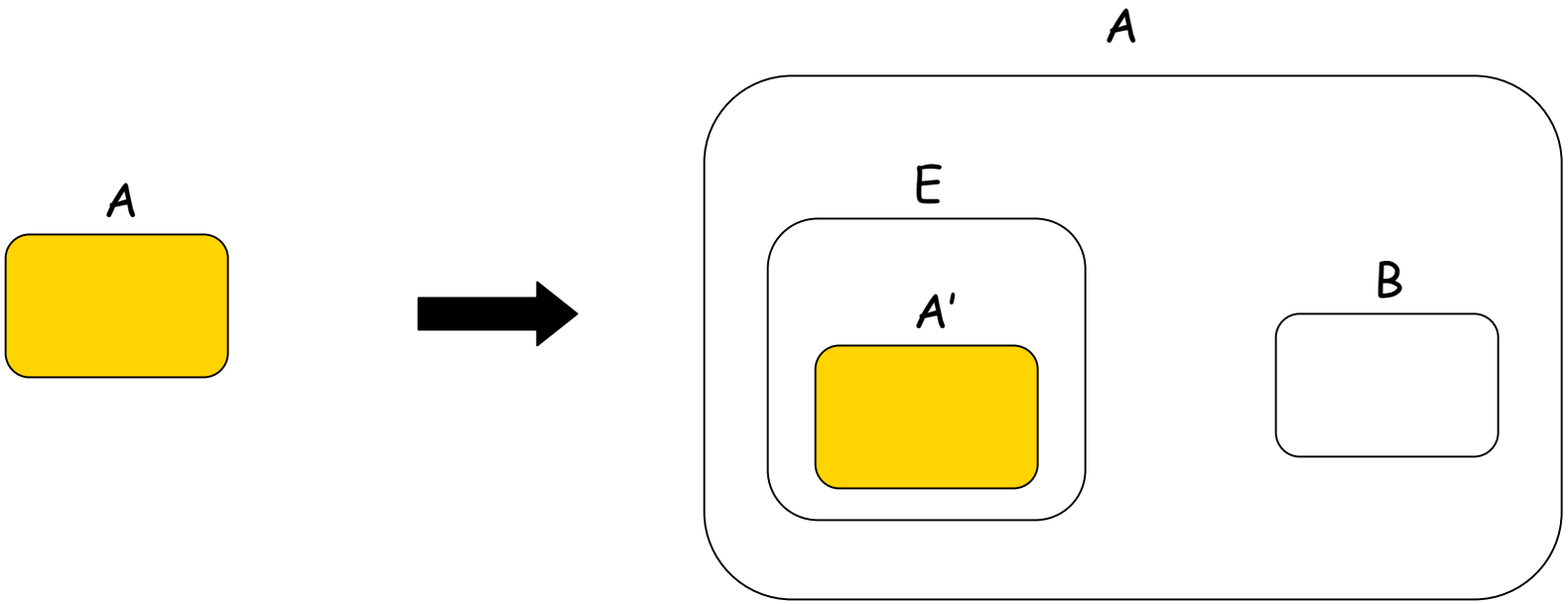
- It's easy to synchronize a fixed number of branes
- Idea: move from a flat membranes structure to a hierarchical structure with growing depth

Initial system



- Each brane encoding object b creates (by dripping) a new brane encoding c
- Each brane encoding object a is surrounded by a newly created brane representing a and containing a new instance of a brane representing b

Evolution of the encoding of object a



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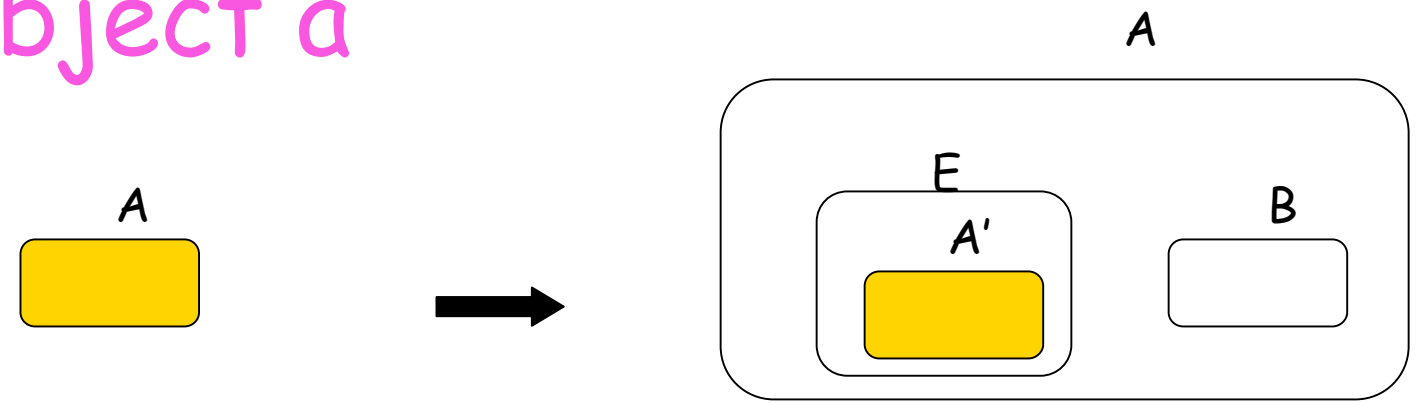
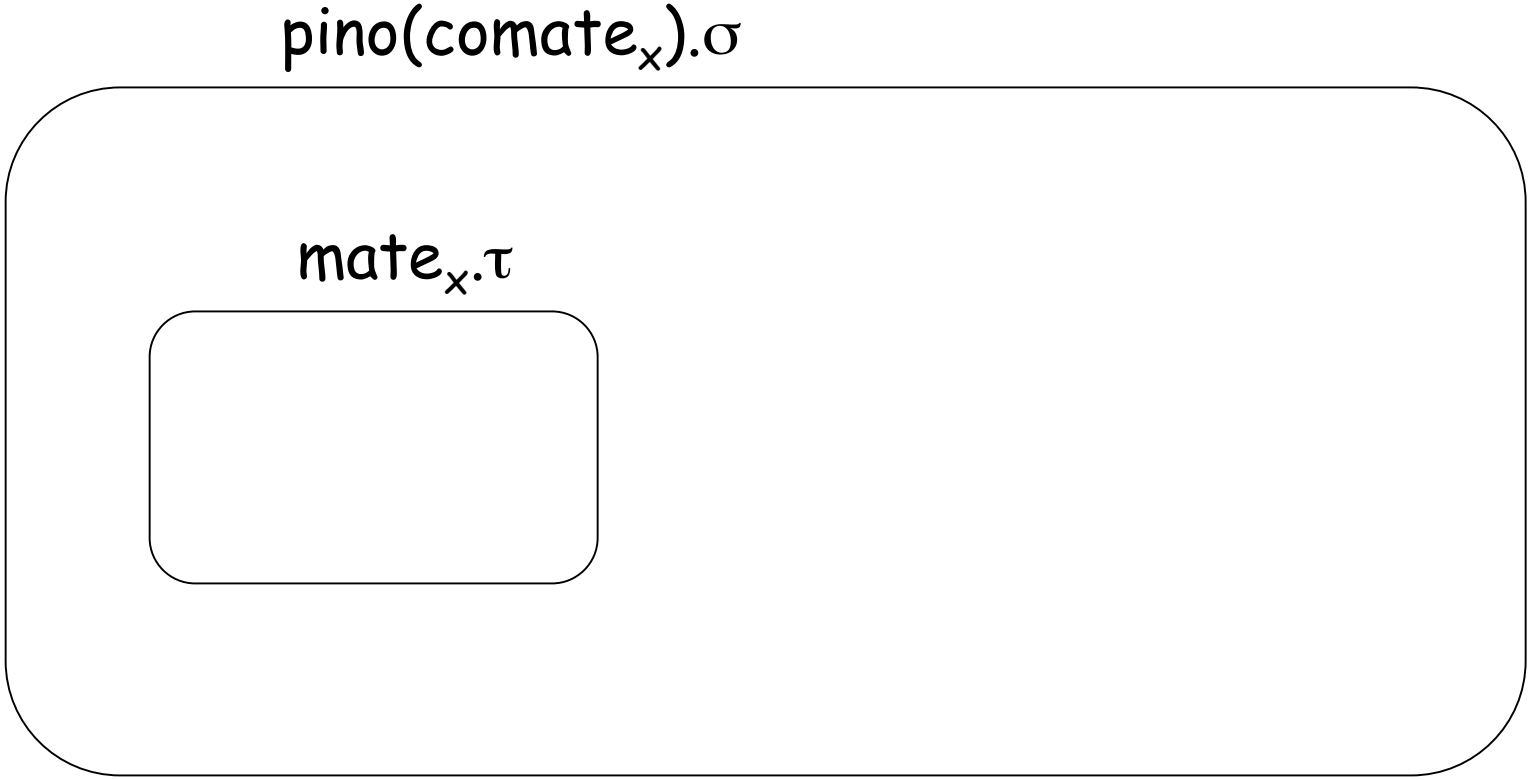


- The A brane sends to signals to its children B and E
  - To wake up instances of B in the hierarchical structure (that will spawn a new C brane)
- The A brane waits for two signals from its children, to ack the end of creation of new copies of C in the hierarchical structure
- A new brane is created; A enters the new bran by phago and spawns a new brane B
- The A brane sends a signal to the external brane to ack the end of its task

Father to child communication



Father to child communication

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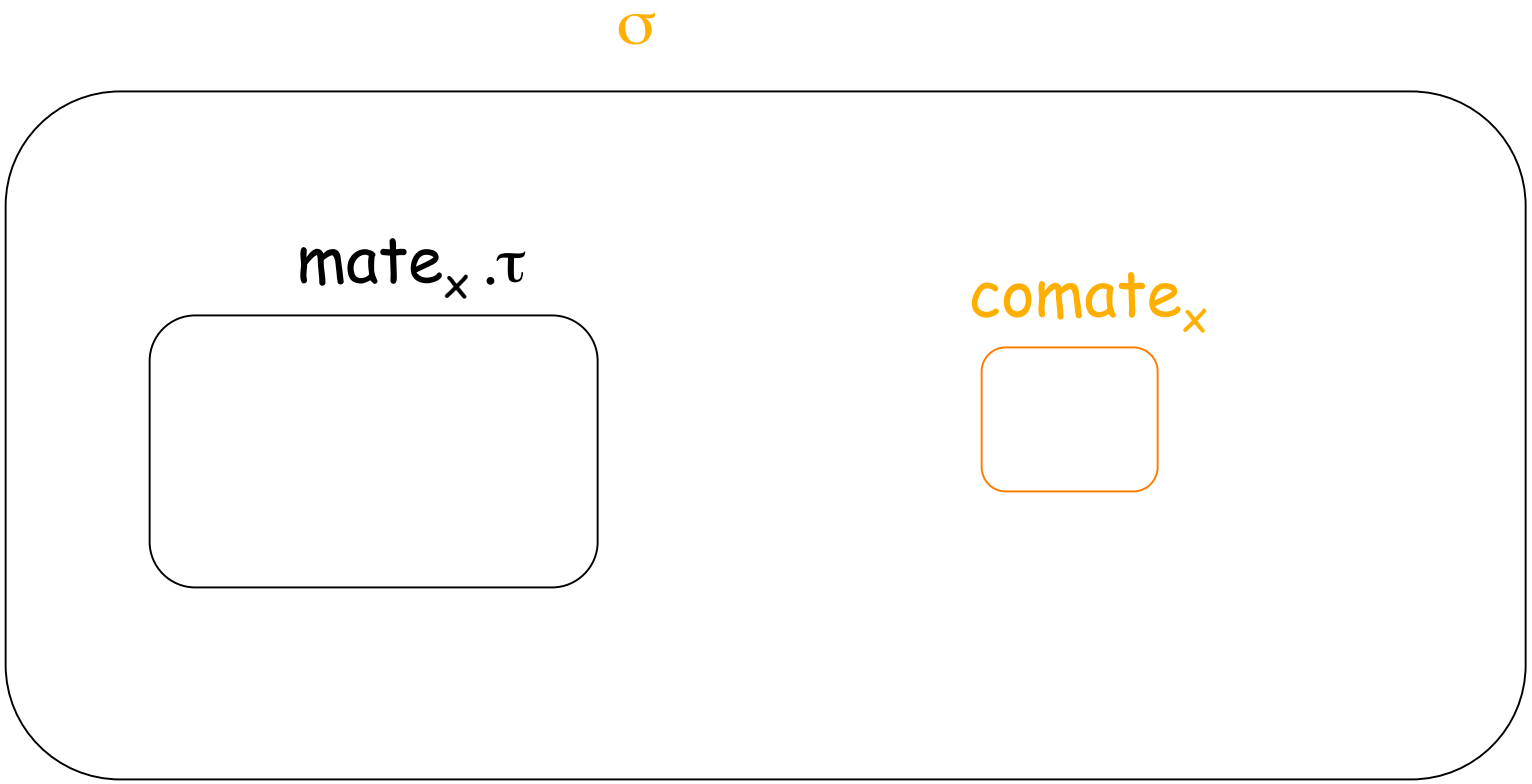

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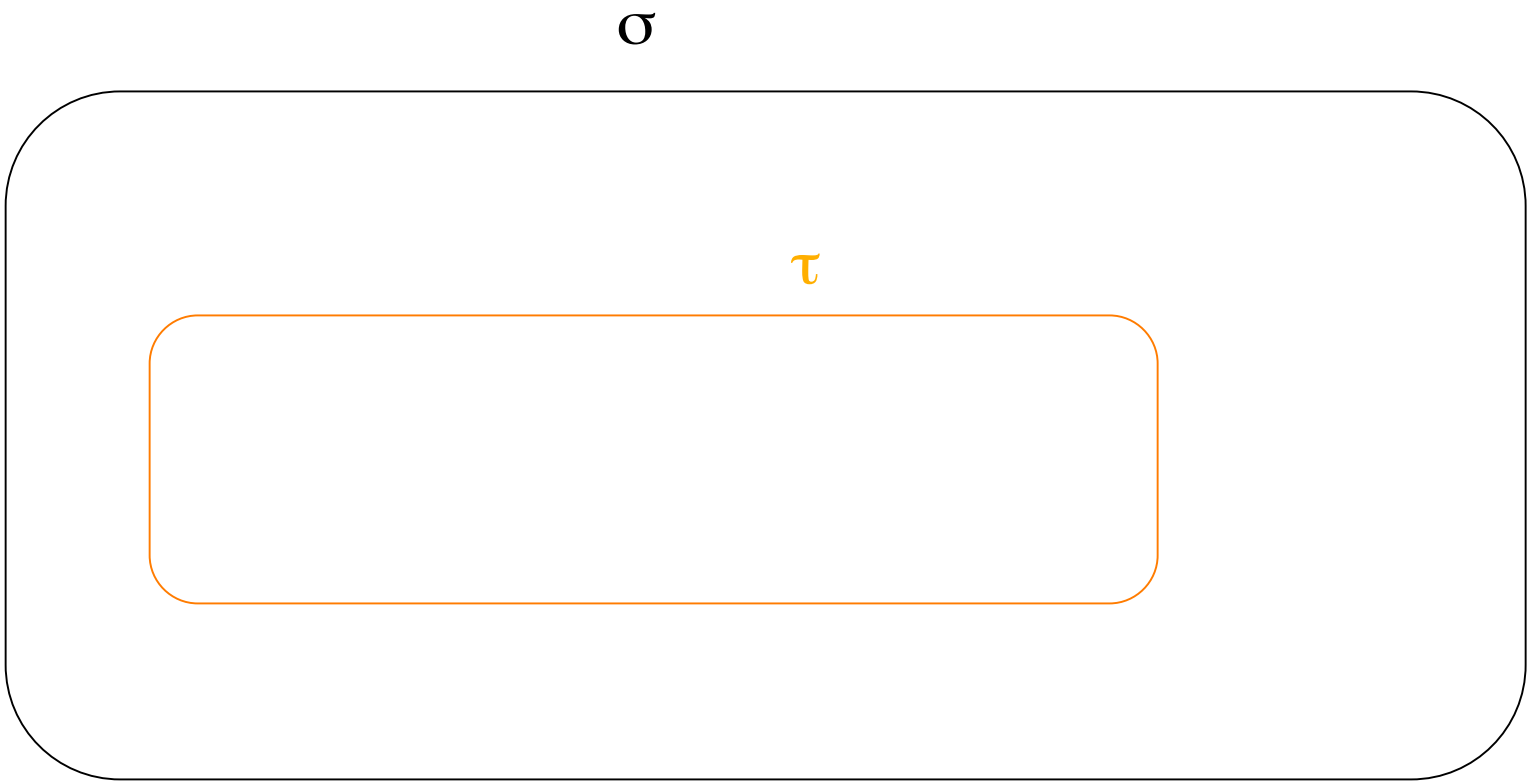

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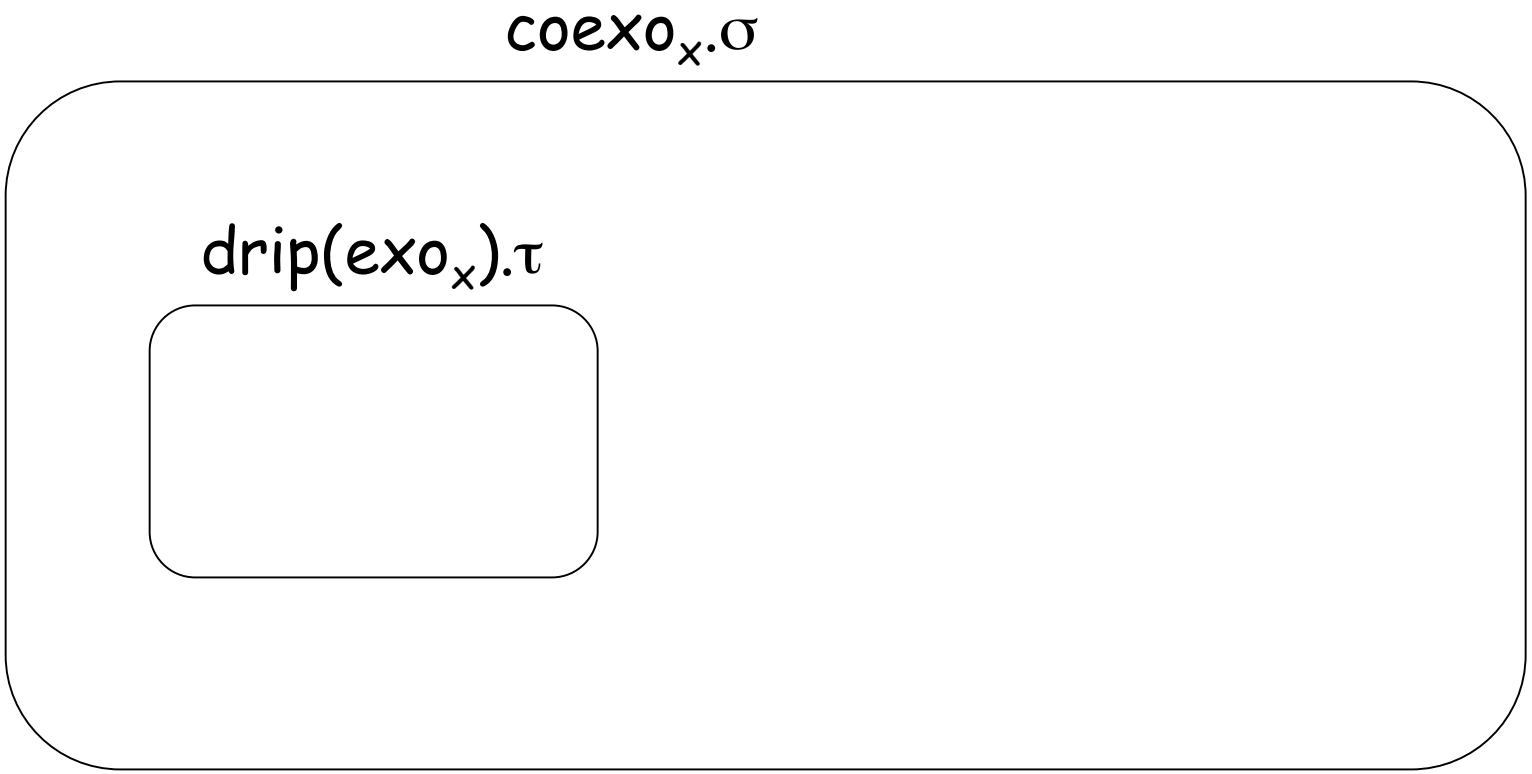

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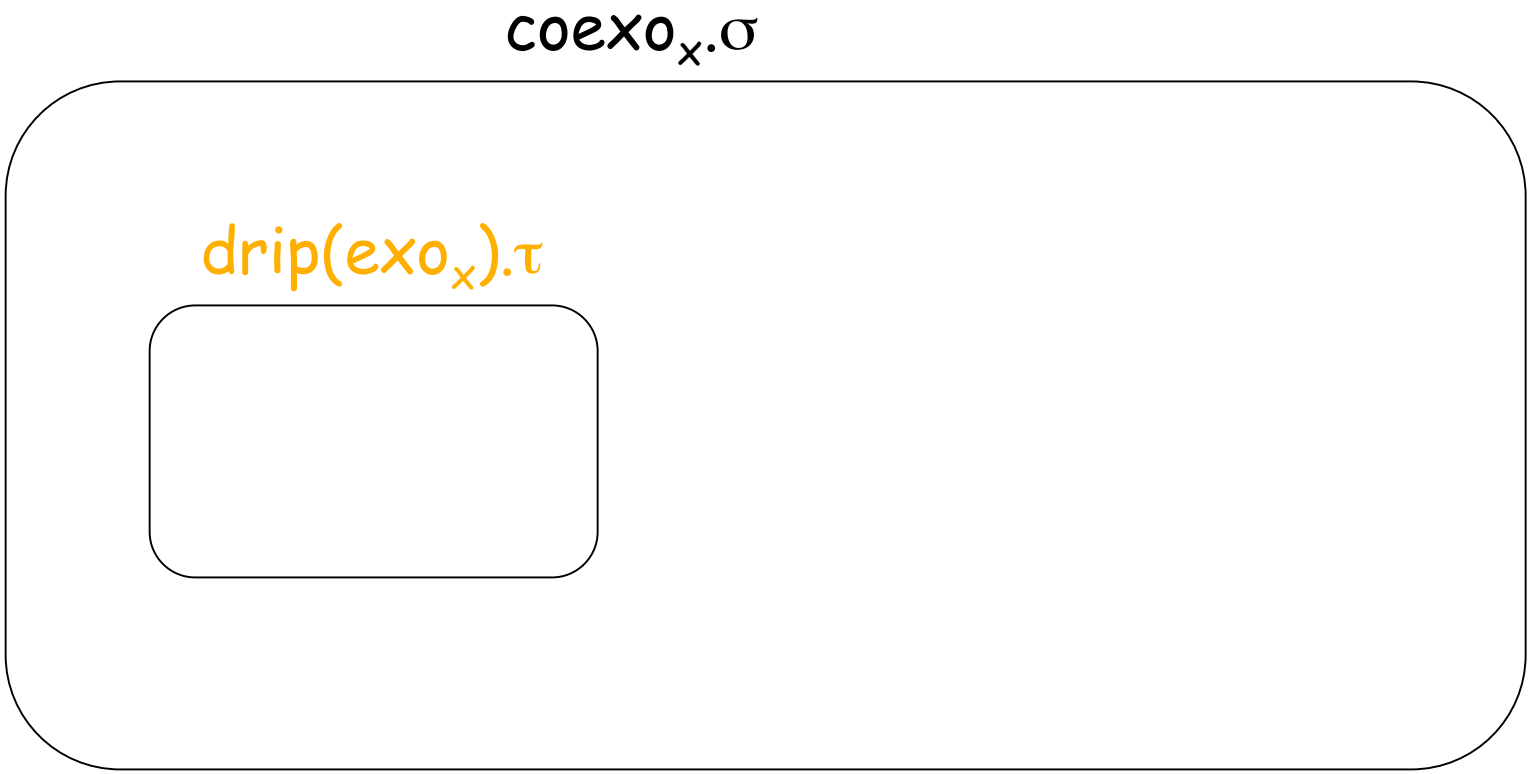

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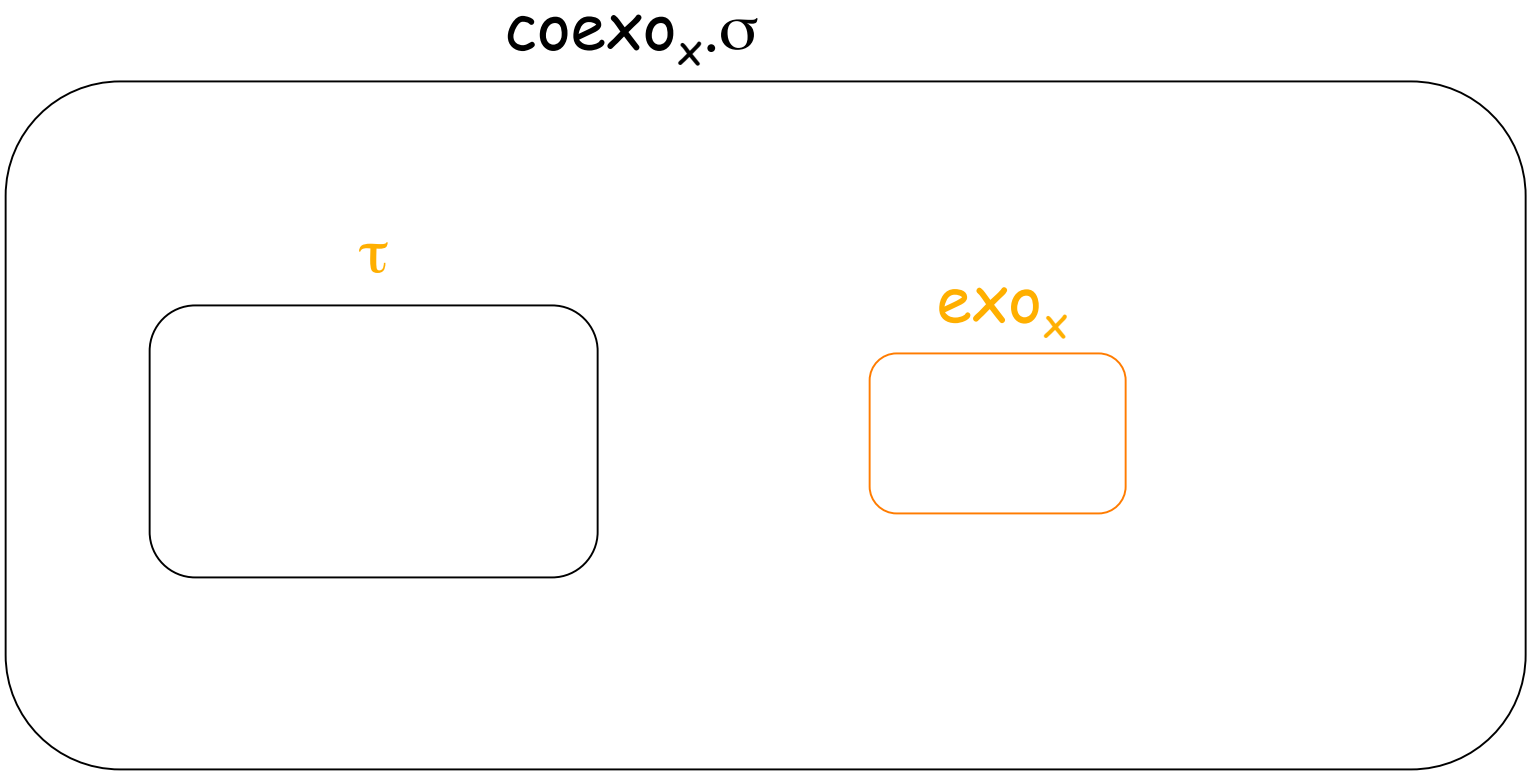

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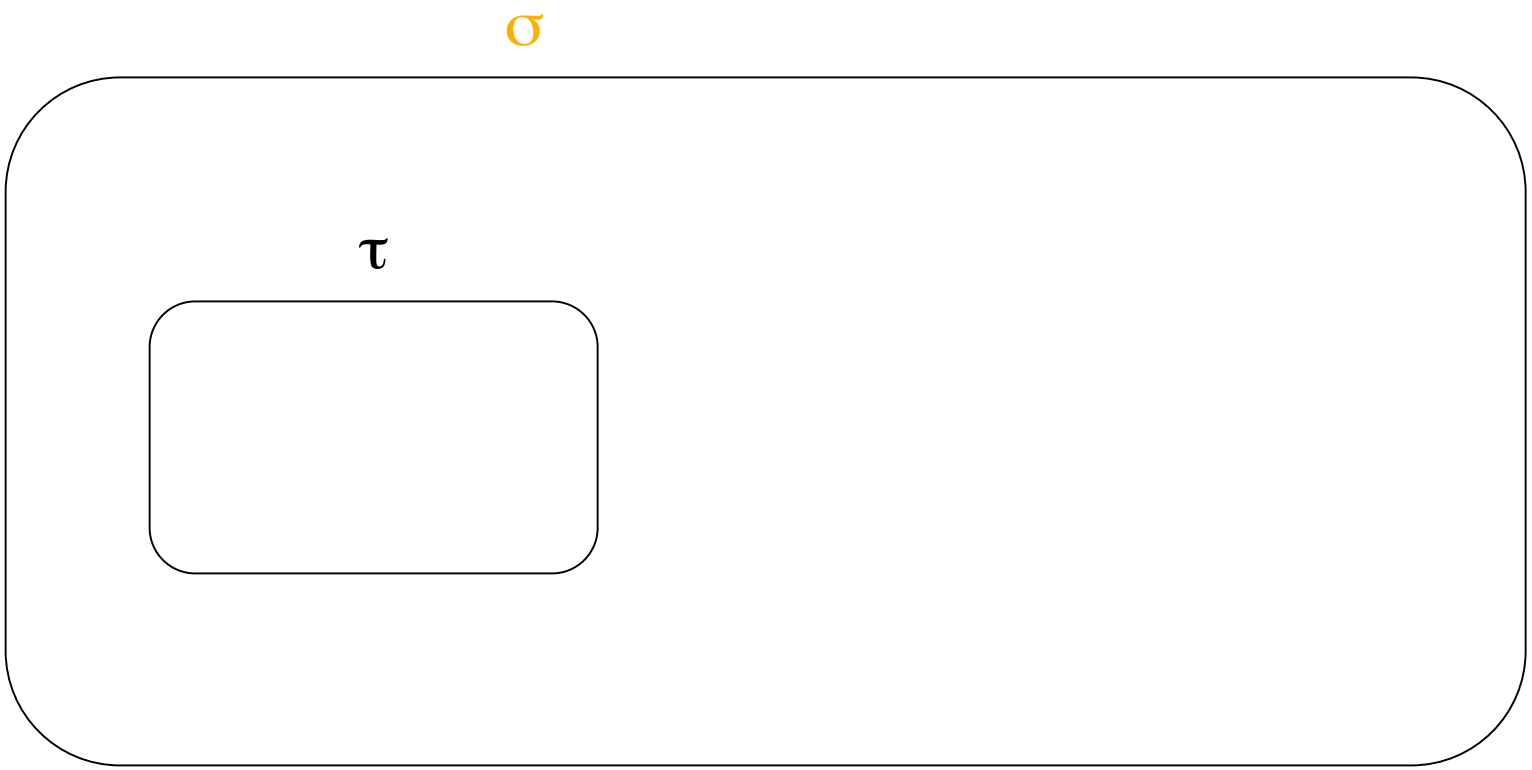

Conclusion

- Semantics (interleaving vs maximal parallelism) is a key difference between P systems and brane calculi
  - See also, e.g., [Freund04, Bus06]
- First step towards polynomial solutions of NP-complete problems in Brane Calculus?

Thank you!